

Enhancing Multi-RAT Coexistence in Unlicensed mmWave Bands Using Hybrid-Beamforming

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Abstract—The radio spectrum is becoming an increasingly scarce and valuable resource to such an extent that sharing the unlicensed bands is inevitable. In this work, we consider a multi-cell, multi-user massive multiple-input multiple-output (MIMO) coexistence scenario where multiple 5G New Radio Unlicensed (NR-U) and Wireless Gigabit (WiGig) links share an unlicensed millimeter-wave band. Our aim is to enhance the performance of coexisting networks by maximizing the overall network throughput via hybrid beamforming. This throughput is a function of both operators' medium access control protocols and physical layer parameters. To maximize the overall network throughput we propose a novel hidden node aware hybrid beamforming design. The hybrid precoders and combiners are optimal in the sense that they simultaneously maximize the signal power at desired users while minimizing the received inter-cell and intra-cell interferences at undesired users (leakage). The performance of the proposed scheme is examined through simulation. A comparison among the proposed method, a beamsteering solution, and an optimal unconstrained precoding design indicates the efficiency of the proposed algorithm.

Index Terms—5G, beamforming, coexistence, hidden nodes, mmWave frequencies, MAC layer, NR-U, physical layer, spectrum sharing, WiGig.

I. INTRODUCTION

Radio spectrum sharing is a key solution to meeting the growing demand for deployed and forth-coming wireless applications and networks. New radio unlicensed (NR-U) extends 5G NR to unlicensed bands as a promising approach to accommodating this growth. A large amount of contiguous bandwidth available at millimeter-wave (mmWave) frequencies can offer multi gigabit-per-second data rates for a myriad of devices [1]. This large amount of bandwidth can be utilized by the NR-U technology to provide mobility and the quality of service that users are expecting from future radio technologies [2]. The opportunity to efficiently utilize the shared and unlicensed spectrum bands, however, leads to the problem of balancing new network paradigms and ensuring coexistence between NR-U and existing networks (e.g., Wireless Gigabit (WiGig) that includes IEEE 802.11ad/ay standard). A similar scenario was faced in the unlicensed 5 GHz band, when coexistence between Long Term Evolution Licensed Assisted Access (LTE-LAA) with IEEE 802.11n Wi-Fi while optimizing key performance indicators (KPIs) of interest was desired.

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The coexistence challenges in the mmWave unlicensed band motivate this work, in which we present a design that can not only enable simultaneous operations of networks within the same mmWave band, but also maximize overall network throughput.

Like LTE-LAA, NR-U uses a listen-before-talk (LBT) mechanism to ensure fair coexistence between different radio access technologies (RATs). LBT is a channel access mechanism by which the station should perform a clear channel assessment (CCA) and backoff to provide channel access fairness and reduce probability of transmission collisions. For NR-U, omni-directional LBT (omniLBT) and directional LBT (dirLBT) have both been considered [3]. Considering dirLBT in NR-U comes from the fact that many NR-U systems will operate at mmWave frequencies, which feature significantly different channel propagation characteristics, and increases the potential for spatial reuse. This difference imposes the use of dense antenna arrays and beam-based transmissions in mmWave frequencies to surmount propagation limits. DirLBT, as its name suggests, senses the medium directionally via beamforming. As opposed to omniLBT, dirLBT has the potential to improve spatial reuse and coexistence of different RATs. However, dirLBT only senses the desired direction and is not aware of the interference coming from other directions. This leads to a change in the interference layout and raises the problem of hidden¹ and exposed² nodes [4]. To complicate the challenge, different RATs utilize different channel sensing thresholds, and transmitter/receiver (TX/RX) coverage ranges.

In a multi-RAT coexistence scenario in which each receiver may suffer from co-channel interference (CCI), eliminating (or mitigating) both intra- and inter-cell interferences is essential. In some cases, linear precoding and power control can be used to mitigate CCI [5]–[7]. However, at mmWave frequencies, beamforming may be more compelling since antenna arrays with a large number of elements are typically implemented at the transmitter and receiver (known as “massive-MIMO”).

¹Hidden nodes happen when two devices cannot “hear” each other as they are out of range of each other. As a result, they start to transmit simultaneously and create mutual interferences at the receiver node, resulting in significantly degraded throughput.

²Exposed node problem occurs when a node stops sending data to a receiver— which is out of range of another one— because of co-channel interference with a neighboring transmitter.

These massive-MIMO arrays also provide the network with more array and multiplexing gain [8]. By combining precoding with beamforming, we can improve the desired signal power while further mitigating potential interference. Utilizing MIMO beamforming is essential to enabling the coexistence of networks since multiple 5G NR Generation NodeBs (gNBs) and user equipments (UEs) are operating on the same unlicensed band as WiGig access points (APs), stations (STAs), and devices.

Recent NR-U and WiGig coexistence research results (e.g., [4], [9]–[11]) have addressed several issues on dirLBT, beamforming, hidden nodes, and system simulation and optimization, beside others. Yet joint beamforming and precoding optimization designs for the transmitter and receiver in a multi-cell multi-RAT mmWave coexistence scenario with consideration of hidden nodes has not been satisfactorily addressed.

In this paper, we consider downlink, full-dimension³ massive MIMO multi-RAT systems in a coexistence scenario. The large number of antennas at each transmitter not only enables beamforming to simultaneously serve a myriad of users but also works to mitigate interference towards other nearby users and networks. By simultaneously allowing each transmitter to make use of spectrum, reuse is maximized, and coexistence is enhanced. We design the hybrid precoders and combiners to maximize network throughput and to support concurrent operation of NR-U and WiGig networks. To calculate the network throughput, we consider both medium access control (MAC) and physical (PHY) layer parameters.

Moreover, existing studies on hybrid beamforming optimization mainly focus on narrowband (i.e., single-subcarrier) massive MIMO systems and they consider either single-cell, single-user or single-cell multi-user MIMO networks [13]–[17]. These studies generally investigate the use of the RF beamforming to selfishly exploit the array gain, while the baseband beamformers are designed to make use of the spatial multiplexing gain. In contrast, we consider a multi-cell, multi-user, multi-stream massive MIMO coexistence scenario in which both NR-U and WiGig networks use wideband orthogonal frequency-division multiple access (OFDMA). We propose a new algorithm which effectively mitigates CCI by designing RF beamformers that reduce the inter-cell interference and the baseband beamformers that alleviate the intra-cell interference, with the aim to increase system throughput.

II. SYSTEM MODEL AND ASSUMPTIONS

The coexistence scenario considered here is a 60 GHz downlink multi-cell, multi-user, massive FD-MIMO situation where two mobile network operators share the same 60 GHz band. To surmount propagation limits, we assume that all transmitters in the network utilize beam-based transmissions. The NR-U network consists of n_{NR} multi-antenna gNBs, while the WiGig network is composed of n_{WiGig} multi-antenna APs.

³Antennas installed in a two-dimensional array provide more degrees of freedom by utilizing both elevation and azimuth domains. This is called “three dimensional massive-MIMO” or “full-dimension MIMO (FD-MIMO)” in 3GPP LTE-Advanced systems [12].

Each transmitter is equipped with N_t transmit antennas and N_t^{RF} RF chains, and plans to communicate with several co-channel associated receivers. Each receiver is also furnished with N_r receive antennas and N_r^{RF} RF chains. The transmitters simultaneously send N_s data symbols per frequency tone to each receiver, which thereby significantly increases the system throughput. In order to decrease the dimension of baseband precoders, reduce the computational complexity at each transmission node, and minimize the channel feedback overhead at the users [18], the RF chains at each transmission node are divided into a number of subsets with M_t^{RF} RF chains in each subset. The number of subsets is equal to the number of users served by a transmission node. For instance, at the transmission node i , $N_t^{\text{RF}} = |\mathcal{U}_i| M_t^{\text{RF}}$ where $|\mathcal{U}_i|$ denotes the number of users associated with the i -th transmission node. Due to limits on spatial multiplexing, and in order to reduce the hardware complexity, the number of RF chains at the transmitter and receiver are subject to $N_s \leq M_t^{\text{RF}} \leq N_t$ and $N_s \leq N_r^{\text{RF}} \leq N_r$, respectively. Geographically, all transmitters are randomly distributed over a particular area, while UEs and STAs are independently and uniformly distributed around each gNB and AP, respectively. It is assumed that both NR-U and WiGig networks are in the saturated traffic condition.

Let $i_k^{(\ell)}$ be the index of the k -th user, $k \in \mathcal{U}_i$, scheduled by the transmission node $i \in \{\mathcal{N}_{\text{NR}}, \mathcal{N}_{\text{WiGig}}\}$ on sub-carrier $\ell \in \mathcal{N}$, where $\mathcal{U}_i = \{1, \dots, |\mathcal{U}_i|\}$, $\mathcal{N}_{\text{NR}} \triangleq \{n_{\text{NR}_1}, n_{\text{NR}_2}, \dots, n_{\text{NR}}\}$, $\mathcal{N}_{\text{WiGig}} \triangleq \{n_{\text{WiGig}_1}, n_{\text{WiGig}_2}, \dots, n_{\text{WiGig}}\}$, $\mathcal{N} \triangleq \{1, 2, \dots, N\}$ and N is the number of sub-carriers. The channel coefficient between the j -th transmitter and the user $i_k^{(\ell)}$ forms channel matrix $\mathbf{H}_{i_k^{(\ell)}, j} = \sqrt{\beta_{i_k^{(\ell)}, j}} \tilde{\mathbf{H}}_{i_k^{(\ell)}, j} \in \mathbb{C}^{N_r \times N_t}$ where $\beta_{i_k^{(\ell)}, j}$ is a large-scale fading coefficient that is used for modeling path-loss, and $\tilde{\mathbf{H}}_{i_k^{(\ell)}, j}$ represents the mmWave channel (delimited by the clustered channel models) between the j -th transmitter and $i_k^{(\ell)}$ user satisfying $\mathbb{E}\{\|\tilde{\mathbf{H}}_{i_k^{(\ell)}, j}\|_F^2\} = N_t N_r$, where expectation is denoted by $\mathbb{E}\{\cdot\}$ and $\|\cdot\|_F$ represents the Frobenius norm. For the user $i_k^{(\ell)}$, the $M_t^{\text{RF}} \times N_s$ and $N_r^{\text{RF}} \times N_s$ low dimensional baseband precoding and combining matrices are denoted as $\mathbf{V}_{i_k^{(\ell)}}^{\text{BB}}$ and $\mathbf{W}_{i_k^{(\ell)}}^{\text{BB}}$, respectively. Moreover, $\mathbf{V}_{i_k^{(\ell)}}^{\text{RF}} \in \mathbb{C}^{N_t \times M_t^{\text{RF}}}$ and $\mathbf{W}_{i_k^{(\ell)}}^{\text{RF}} \in \mathbb{C}^{N_r \times N_r^{\text{RF}}}$ indicate the RF precoding and combining matrices, respectively.

Following the above notation and assuming that all the signals from the transmitters arrive at the user $i_k^{(\ell)}$ synchronously, the final processed signal at this user is given by (1), where $\rho_{i_k^{(\ell)}, i}$ represents the average received power at user $i_k^{(\ell)}$, $\mathbf{s}_{i_k^{(\ell)}}$ denotes the desired transmitted signal for user $i_k^{(\ell)}$ with $\mathbb{E}\{\mathbf{s}_{i_k^{(\ell)}} \mathbf{s}_{i_k^{(\ell)}}^H\} = 1/N_s \mathbf{I}_{N_s}$, and $\mathbf{n}_{i_k^{(\ell)}} \sim \mathcal{CN}(\mathbf{0}, \sigma_{i_k^{(\ell)}}^2 \mathbf{I}_{N_r})$ is the additive white Gaussian noise on the ℓ -th sub-carrier. Assuming that the users can share each sub-carrier in a time sharing manner and there are no hidden nodes in the network, the downlink achievable data rate of user $i_k^{(\ell)}$ can be expressed as (2), where $0 \leq \eta_i \leq 1$ is the fraction of time that the unlicensed channel is being used by the i -th transmission node and can be represented as the successful

$$\hat{\mathbf{s}}_{i_k}^{(\ell)} = \underbrace{\sqrt{\rho_{i_k}^{(\ell)}} \mathbf{W}_{i_k}^{\text{BBH}} \mathbf{W}_{i_k}^{\text{RFH}} \tilde{\mathbf{H}}_{i_k}^{(\ell)} \mathbf{V}_{i_k}^{\text{RF}} \mathbf{V}_{i_k}^{\text{BB}} \mathbf{s}_{i_k}^{(\ell)}}_{\text{Desired signal}} + \underbrace{\sum_{m=1, m \neq k}^{|\mathcal{U}_i|} \sqrt{\rho_{i_m}^{(\ell)}} \mathbf{W}_{i_m}^{\text{BBH}} \mathbf{W}_{i_m}^{\text{RFH}} \tilde{\mathbf{H}}_{i_m}^{(\ell)} \mathbf{V}_{i_m}^{\text{RF}} \mathbf{V}_{i_m}^{\text{BB}} \mathbf{s}_{i_m}^{(\ell)}}_{\text{Intra-cell interference}} + \underbrace{\sum_{j=1, j \neq i}^{n_{\text{NR}} + n_{\text{WiGig}}} \sum_{m=1}^{|\mathcal{U}_j|} \sqrt{\rho_{j_m}^{(\ell)}} \mathbf{W}_{j_m}^{\text{BBH}} \mathbf{W}_{j_m}^{\text{RFH}} \tilde{\mathbf{H}}_{j_m}^{(\ell)} \mathbf{V}_{j_m}^{\text{RF}} \mathbf{V}_{j_m}^{\text{BB}} \mathbf{s}_{j_m}^{(\ell)}}_{\text{Inter-cell interference}} + \underbrace{\mathbf{W}_{i_k}^{\text{BBH}} \mathbf{W}_{i_k}^{\text{RFH}} \mathbf{n}_{i_k}^{(\ell)}}_{\text{Noise}} \quad (1)$$

transmission probability of the i -th transmission node, $0 \leq \gamma_{i_k}^{(\ell)} \leq 1$ is the fraction of time allotted to the user $i_k^{(\ell)}$ at the ℓ -th subcarrier on the unlicensed band that satisfies $0 \leq \sum_k \gamma_{i_k}^{(\ell)} \leq 1$, B is the sub-carrier's bandwidth, and $\mathbf{Q}_{i_k}^{(\ell)} = \mathbf{H}_{i_k}^{(\ell), i} \mathbf{V}_{i_k}^{(\ell)} \mathbf{V}_{i_k}^{\text{H}} \mathbf{H}_{i_k}^{\text{H}} \mathbf{H}_{i_k}^{(\ell), i}$ and $\mathbf{D}_{i_k}^{(\ell)} = \mathbf{W}_{i_k}^{\text{H}} (\sigma^2 \mathbf{I} + \sum_{(m,j) \neq (k,i)} \rho_{j_m}^{(\ell)} \mathbf{H}_{i_k}^{(\ell), j} \mathbf{V}_{j_m}^{(\ell)} \mathbf{V}_{j_m}^{\text{H}} \mathbf{H}_{i_k}^{\text{H}} \mathbf{H}_{i_k}^{(\ell), j}) \mathbf{W}_{i_k}^{(\ell)}$ represent the covariance matrix of the desired signal and the interference-plus-noise covariance matrix associated with user $i_k^{(\ell)}$, respectively ($\mathbf{V}_{i_k}^{(\ell)} \triangleq \mathbf{V}_{i_k}^{\text{RF}} \mathbf{V}_{i_k}^{\text{BB}}$, $\mathbf{W}_{i_k}^{(\ell)} \triangleq \mathbf{W}_{i_k}^{\text{RF}} \mathbf{W}_{i_k}^{\text{BB}}$).

III. THE IMPACT OF THE HIDDEN NODES ON THE NETWORK THROUGHPUT

If hidden nodes are present in the network, they may cause interference to other nodes, and potentially decrease the network throughput. Incorporating the effect of hidden nodes into calculating the achievable data rate can enable studies to better mitigate interference among transmission nodes. As a result, the transmission nodes can fully utilize the unlicensed band, resulting in improved total network throughput.

To analyze this, effect let us assume $p_{i_k}^{\text{HN}}$ indicates the probability that user $i_k^{(\ell)}$ is affected by hidden nodes from another operator, i.e., due to the interference from hidden nodes the physical layer header is corrupted. Conforming to the analytical model in [19], the probability that transmission node i successfully receives an acknowledgment frame from user $i_k^{(\ell)}$ can be expressed as

$$p_i^{\text{ACK}} = (1 - p_{c,i})(1 - p_{i_k}^{\text{HN}}) + p_{c,i} p_i^{\text{cap}}, \quad (3)$$

where $p_{c,i}$ is probability that at least one node among the other transmission nodes is transmitting with the transmission node i at the same time slot and can be calculated as follows

$$p_{c,i} = \begin{cases} 1 - \prod_{j \in \mathcal{N}_{\text{WiGig}} / \{i\}} (1 - p_{\text{tr},j}) \prod_{\ell \in \mathcal{N}_{\text{NR}}} (1 - p_{\text{tr},\ell}), & \text{if } i \in \mathcal{N}_{\text{WiGig}} \\ 1 - \prod_{w \in \mathcal{N}_{\text{WiGig}}} (1 - p_{\text{tr},w}) \prod_{j \in \mathcal{N}_{\text{NR}} / \{i\}} (1 - p_{\text{tr},j}), & \text{if } i \in \mathcal{N}_{\text{NR}} \end{cases}$$

where $p_{\text{tr},i}$ indicates the packet transmission probability of the i -th transmission node and can be written as

$$R_{i_k}^{(\ell)} = \begin{cases} \eta_i \gamma_{i_k}^{(\ell)} B \log_2 \det \left(\mathbf{I}_{N_s} + \frac{\rho_{i_k}^{(\ell)}}{N_s} \mathbf{W}_{i_k}^{\text{BBH}} \mathbf{W}_{i_k}^{\text{RFH}} \mathbf{Q}_{i_k}^{(\ell)} \mathbf{W}_{i_k}^{\text{RF}} \mathbf{W}_{i_k}^{\text{BB}} \mathbf{D}_{i_k}^{-1} \right), & \eta_i > 0, \gamma_{i_k}^{(\ell)} > 0 \\ 0, & \eta_i = 0, \gamma_{i_k}^{(\ell)} = 0 \end{cases} \quad (2)$$

$$p_{\text{tr},i} = \frac{2(1-2p_{c,i})}{(1-2p_{c,i})(1+\text{CW}_{\min,i}) + p_{c,i} \text{CW}_{\min,i}(1-(2p_{c,i})^{m_i})},$$

where the minimum contention window size and the maximum back-off stage of the i -th transmission node are indicated respectively by $\text{CW}_{\min,i}$ and m_i . It is worth noting that not all of these simultaneous transmissions by the other transmission nodes in the carrier sensing span of the transmission node i results in a collision. To be specific, if the received SINR at user $i_k^{(\ell)}$ is larger than the predefined threshold, user $i_k^{(\ell)}$ can still decode the data. This is called the capture effect [20] and is represented by p_i^{cap} in (3). Note that due to the dependency of p_i^{cap} on the comparative location of the transmission node i and the other concurrent transmission nodes (that is subject to change with time), estimating the value of p_i^{cap} is hard to accomplish. Therefore, for simplicity, we acquire a lower bound of $p_{i_k}^{\text{HN}}$ by setting $p_i^{\text{cap}} = 0$ as in [19]. Note that the probability that the unlicensed band is being occupied by either NR-U or WiGig transmission can be calculated as

$$p_{\text{tr}} = 1 - \prod_{i \in \mathcal{N}_{\text{NR}}} (1 - p_{\text{tr},i}) \prod_{j \in \mathcal{N}_{\text{WiGig}}} (1 - p_{\text{tr},j}).$$

Consequently, $p_{c,i}$ can be calculated as a function of p_{tr} , i.e., $p_{c,i} = 1 - (1 - p_{\text{tr}})/(1 - p_{\text{tr},i})$, and $p_{i_k}^{\text{HN}}$ can be obtained as

$$p_{i_k}^{\text{HN}} = 1 - \frac{p_i^{\text{ACK}}(1 - p_{\text{tr},i})}{(1 - p_{\text{tr}})}, \quad (4)$$

where all of these parameters can be obtained using measurable MAC layer statistics [5], [21].

We assume both NR-U and WiGig networks have predefined thresholds $p_{\text{NR-U}}^{\text{th}}$ and $p_{\text{WiGig}}^{\text{th}}$ to tolerate hidden nodes, respectively. At each network, when the probability that a user is being affected by hidden nodes exceeds this predefined threshold, the user suffers severe interference from hidden nodes and the LBT mechanism is likely to fail, causing significantly lower network throughput. Here, we assume the user's achievable downlink rate is zero. Otherwise, the achievable downlink rate of the user $i_k^{(\ell)}$ in the unlicensed band can be expressed as $(1 - p_{i_k}^{\text{HN}}) R_{i_k}^{(\ell)}$.

IV. HIDDEN NODE AWARE BEAMFORMING DESIGN

The problem of interest is to maximize the overall network throughput of networks operating in this unlicensed band over the hidden node aware resource management and allocation matrices (hybrid precoders/combiners), i.e.,

$$\begin{aligned} \mathcal{P}_1 : \quad & \underset{\{\mathbf{V}_{i_k}^{\text{RF}}\}, \{\mathbf{W}_{i_k}^{\text{RF}}\}}{\{\mathbf{V}_{i_k}^{\text{BB}}\}, \{\mathbf{W}_{i_k}^{\text{BB}}\}}}{\text{maximize}} \quad S_{\mathcal{N}}(\mathbf{V}_{i_k}^{(\ell)}, \mathbf{W}_{i_k}^{(\ell)}) + S_{\mathcal{W}}(\mathbf{V}_{i_k}^{(\ell)}, \mathbf{W}_{i_k}^{(\ell)}) \\ & \text{subject to} \quad \|\mathbf{V}_{i_k}^{\text{RF}} \mathbf{V}_{i_k}^{\text{BB}} \mathbf{P}_{i_k}^{(\ell)}\|_F^2 = N_s \\ & \quad |\mathbf{V}_{i_k}^{\text{RF}}| = 1/\sqrt{N_t} \\ & \quad |\mathbf{W}_{i_k}^{\text{RF}}| = 1/\sqrt{N_r} \end{aligned}$$

where $S_{\mathcal{N}}$ and $S_{\mathcal{W}}$ are the networks throughput of NR-U and WiGig systems, respectively, and are functions of both MAC and PHY layer parameters [21], can be written as

$$\begin{aligned} S_{\mathcal{N}} &= p_{s,\mathcal{N}} T_{P,\mathcal{N}} R_{\mathcal{N}} / T_{\text{avg}}, \\ S_{\mathcal{W}} &= p_{s,\mathcal{W}} T_{P,\mathcal{W}} R_{\mathcal{W}} / T_{\text{avg}}, \end{aligned}$$

where $p_{s,\mathcal{N}}$ and $p_{s,\mathcal{W}}$ stand for the successful transmission probability of the entire NR-U and WiGig networks, respectively, and can be calculated as

$$\begin{aligned} p_{s,\mathcal{N}} &= \sum_{i \in \mathcal{N}_{\text{NR}}} p_{\text{tr},i} \prod_{w \in \mathcal{N}_{\text{WiGig}}} (1 - p_{\text{tr},w}) \prod_{j \in \mathcal{N}_{\text{NR}}/\{i\}} (1 - p_{\text{tr},j}), \\ p_{s,\mathcal{W}} &= \sum_{i \in \mathcal{N}_{\text{WiGig}}} p_{\text{tr},i} \prod_{j \in \mathcal{N}_{\text{WiGig}}/\{i\}} (1 - p_{\text{tr},j}) \prod_{\ell \in \mathcal{N}_{\text{NR}}} (1 - p_{\text{tr},\ell}), \end{aligned}$$

$T_{P,\mathcal{N}}$ and $T_{P,\mathcal{W}}$ indicate the NR-U and WiGig payload duration, respectively, and $R_{\mathcal{N}}$ and $R_{\mathcal{W}}$ refer to the NR-U's and WiGig's physical achievable data rates, respectively, and can be obtained as follows

$$R = \sum_i \sum_k \sum_{\ell} (1 - p_{i_k}^{\text{HN}}) R_{i_k}^{(\ell)}.$$

If $i \in \mathcal{N}_{\text{NR}}$, $R = R_{\mathcal{N}}$. Otherwise $R = R_{\mathcal{W}}$. The T_{avg} denotes the average time duration to assist a successful transmission in the network, and can be calculated based on the measurable MAC layer statistics as in [21].

The optimization problem \mathcal{P}_1 is a joint (multi-criteria) optimization problem over both analog RF and digital baseband precoding/combining matrices. Obtaining the global optimal solution for similar constrained joint optimization problems is intractable [22], [23]. At mmWave frequencies, the non-convex constraint on the RF precoders and combiners makes obtaining an optimal solution even more difficult. To simplify this design and reduce the downlink training overhead while considering the practical constraints on RF hardware, we decouple this joint optimization problem into the analog precoder/combiner part and the digital counterpart as in [18], [24].

In the proposed design, the precoders/combiners are designed in such a way that not only regulate the transmit beam of each user to construct an aligned directional beam that enhances the strength of the desired signal but also minimizes the received interference signal power towards undesired users. This goal can be achieved by maximizing the received SINR for each user. However, because of the coupled nature of

this problem, solving it requires joint operation from almost all aspects of MIMO communication. There is essentially no closed-form solution available for this optimization problem. As an alternative ‘‘optimization metric’’ the authors in [25] proposed a new metric, called signal-to-leakage-and-noise ratio (SLNR), that transforms the coupled optimization problem of maximizing SINR into a completely decoupled one with an available closed-form solution. The new optimization problem does not impose any restriction either on the RF hardware or on the number of users and data streams in contrast to the block diagonalization scheme [26] or the coordinated beamforming algorithm [27], respectively. Hence, we adopt this ‘‘optimization metric’’ to transfer the original optimization problem to a tractable but sub-optimal precoders/combiners design [28], [29], i.e., maximizing SLNR instead of maximizing SINR, to enhance the strength of the desired signal while minimizing/mitigating the residual interference towards undesired users.

Utilizing the SLNR metric in our design, the SLNR of the serving user $i_k^{(\ell)}$ can be written as

$$\text{SLNR}_{i_k^{(\ell)}}^{\text{inter}} = \frac{\text{trace}(\mathbf{V}_{i_k}^{\text{RFH}} \mathbf{H}_{i_k,i}^{\text{H}} \mathbf{H}_{i_k,i} \mathbf{V}_{i_k}^{\text{RF}})}{\text{trace}(\mathbf{V}_{i_k}^{\text{RFH}} [\sigma^2 \mathbf{I}_{N_t} + \bar{\mathbf{H}}_{i_k}^{\text{interH}} \bar{\mathbf{H}}_{i_k}^{\text{inter}}] \mathbf{V}_{i_k}^{\text{RF}})}$$

where

$$\begin{aligned} \bar{\mathbf{H}}_{i_k}^{\text{inter}} &= \underbrace{[\mathbf{H}_{1,i}^{\text{H}}, \dots, \mathbf{H}_{1,|U_1|,i}^{\text{H}}, \dots, \mathbf{H}_{i-1,i}^{\text{H}}, \dots, \mathbf{H}_{i-1,|U_{i-1}|,i}^{\text{H}}]}_{\text{cell } 1} \underbrace{[\mathbf{H}_{i+1,i}^{\text{H}}, \dots, \mathbf{H}_{i+1,|U_{i+1}|,i}^{\text{H}}, \dots, \mathbf{H}_{G,i}^{\text{H}}, \dots, \mathbf{H}_{G,|U_G|,i}^{\text{H}}]}_{\text{cell } i+1 \text{ to } G} \end{aligned}$$

represents the concatenated inter-cell leakage channel corresponds to the channel from the i -th transmission node to the $i_k^{(\ell)}$ and $G = n_{\text{NR}} + n_{\text{WiGig}}$. The RF precoders are aimed at maximizing the SLNR and designed such that

$$\mathbf{V}_{i_k}^{\text{RF}*} = \underset{\mathbf{V}_{i_k}^{\text{RF}} \in \mathbb{C}^{N_t \times M_t^{\text{RF}}}}{\text{arg max}} \quad \text{SLNR}_{i_k^{(\ell)}},$$

with $\text{trace}(\mathbf{V}_{i_k}^{\text{RF}} \mathbf{V}_{i_k}^{\text{RFH}}) = N_s$ for power limitation. Hence, the optimal RF precoders (in the sense of SLNR maximization) can be acquired by extracting the leading M_t^{RF} columns of the generalized eigenmatrix of the pair $(\mathbf{H}_{i_k,i}^{\text{H}} \mathbf{H}_{i_k,i}, \sigma^2 \mathbf{I}_{N_t} + \bar{\mathbf{H}}_{i_k}^{\text{interH}} \bar{\mathbf{H}}_{i_k}^{\text{inter}})$ [30], namely $\mathbf{T}_{i_k^{(\ell)}}$, as

$$\mathbf{V}_{i_k}^{\text{RF}*} = \mu \mathbf{T}_{i_k^{(\ell)}} [\mathbf{I}_{M_t^{\text{RF}}}; \mathbf{0}], \quad (5)$$

where μ indicates a scaling factor to fulfill the power limitation. Note that the optimal solution given in (5) does not satisfy the constraint on the RF precoders, i.e., constant module. To satisfy this constraint we represent the optimal RF precoder by $\hat{\mathbf{V}}_{i_k}^{\text{RF}*}$ which aims at minimizing $\|\hat{\mathbf{V}}_{i_k}^{\text{RF}*} - \mathbf{V}_{i_k}^{\text{RF}*}\|_2^2$. It is easy to show that this goal achieves by maximizing $\text{Real}(\mathbf{V}_{i_k}^{\text{RF}*H} \hat{\mathbf{V}}_{i_k}^{\text{RF}*})$ which happens when

$$\hat{\mathbf{V}}_{i_k}^{\text{RF}*} = \frac{1}{\sqrt{N_t}} \exp(j \mathbf{V}_{i_k}^{\text{RF}*}). \quad (6)$$

Given the optimal RF precoding matrices by (6), the RF combiners are designed such that the total inter-cell interference leakage, i.e., interference due to all undesired transmission nodes, at the user $i_k^{(\ell)}$ gets minimized, i.e.,

$$\mathbf{W}_{i_k^{(\ell)}}^{\text{RF}*} = \arg \min_{\mathbf{W}_{i_k^{(\ell)}}^{\text{RF}} \in \mathbb{C}^{N_r \times N_r^{\text{RF}}}} \text{trace}(\mathbf{W}_{i_k^{(\ell)}}^{\text{RFH}} \mathbf{R}_{i_k^{(\ell)}} \mathbf{W}_{i_k^{(\ell)}}^{\text{RF}}),$$

where $\mathbf{R}_{i_k^{(\ell)}}$ denotes the inter-cell interference covariance matrix at the user $i_k^{(\ell)}$ and can be calculated as follows

$$\mathbf{R}_{i_k^{(\ell)}} = \sum_{j \neq i} \sum_m \mathbf{H}_{i_k^{(\ell)}, j} \hat{\mathbf{V}}_{j_m^{(\ell)}}^{\text{RF}*} \hat{\mathbf{V}}_{j_m^{(\ell)}}^{\text{RFH}} \mathbf{H}_{i_k^{(\ell)}, j}^{\text{H}}. \quad (7)$$

Hence, the b -th column of the optimal RF combiner $\mathbf{W}_{i_k^{(\ell)}}^{\text{RF}*}$ is commensurate with the b -th smallest eigenvalue of $\mathbf{R}_{i_k^{(\ell)}}$, i.e.,

$$\mathbf{W}_{i_k^{(\ell)}}^{\text{RF}*}(:, b) = \nu_b(\mathbf{R}_{i_k^{(\ell)}}), \quad (8)$$

where $\nu_b(\mathbf{A})$ is the eigenvector corresponding to the b -th smallest eigenvalue of \mathbf{A} . The physical meaning of this solution is the space extended across the eigenvectors representing the N_r^{RF} smallest eigenvalues of the interference covariance matrix $\mathbf{R}_{i_k^{(\ell)}}$ is the N_r^{RF} dimensional received signal subspace that consists of the least interference. To satisfy the constant module constraints the optimal RF combiners should be modified as follows

$$\hat{\mathbf{W}}_{i_k^{(\ell)}}^{\text{RF}*} = \frac{1}{\sqrt{N_r}} \exp(j \mathbf{W}_{i_k^{(\ell)}}^{\text{RF}*}). \quad (9)$$

After obtaining and applying the optimal RF precoders and combiners, the inter-cell interference in the i -th cell is minimized. These matrices, however, do not guarantee the suppression of the interference between multi-user data streams in the i -th cell. Hence, design of the baseband counterparts should be done such that intra-cell interference is eliminated. To do so, we invoke the optimal RF precoders and combiners in (1) and define the concatenated intra-leakage channel matrix corresponds to the channel from the transmission node i to the user $i_k^{(\ell)}$ as

$$\bar{\mathbf{H}}_{i_k}^{\text{intra}} = [\mathbf{H}_{i_1^{(\ell)}, i_k}^{\text{EffH}}, \dots, \mathbf{H}_{i_{k-1}^{(\ell)}, i_k}^{\text{EffH}}, \mathbf{H}_{i_{k+1}^{(\ell)}, i_k}^{\text{EffH}}, \dots, \mathbf{H}_{i_{|\mathcal{U}_i|}^{(\ell)}, i_k}^{\text{EffH}}]^{\text{H}},$$

where the effective channel matrix is expressed as $\mathbf{H}_{i_k^{(\ell)}, j_m^{(\ell)}}^{\text{Eff}} = \mathbf{W}_{i_k^{(\ell)}}^{\text{RFH}} \mathbf{H}_{i_k^{(\ell)}, j} \mathbf{V}_{j_m^{(\ell)}}^{\text{RF}}$. Therefore, the SLNR at the receiver $i_k^{(\ell)}$ can be written as

$$\text{SLNR}_{i_k^{(\ell)}}^{\text{intra}} = \frac{\text{trace}(\mathbf{V}_{i_k^{(\ell)}}^{\text{BBH}} \mathbf{H}_{i_k^{(\ell)}, i_k}^{\text{EffH}} \mathbf{H}_{i_k^{(\ell)}, i_k}^{\text{Eff}} \mathbf{V}_{i_k^{(\ell)}}^{\text{BB}})}{\text{trace}(\mathbf{V}_{i_k^{(\ell)}}^{\text{BBH}} [\tilde{\sigma}_{i_k^{(\ell)}}^2 \mathbf{I}_{M_t^{\text{RF}}} + \bar{\mathbf{H}}_{i_k^{(\ell)}}^{\text{intraH}} \bar{\mathbf{H}}_{i_k^{(\ell)}}^{\text{intra}}] \mathbf{V}_{i_k^{(\ell)}}^{\text{BB}})},$$

where $\tilde{\sigma}_{i_k^{(\ell)}}^2 = \sigma_{i_k^{(\ell)}}^2 + \rho_{N_r^{\text{RF}}}(\mathbf{R}_{i_k^{(\ell)}})$ that $\rho_n(\mathbf{A})$ is the sum of n smallest eigenvalues of \mathbf{A} . Following the same approach, the optimal baseband precoder is obtained as

$$\mathbf{V}_{i_k^{(\ell)}}^{\text{BB}*} = \tilde{\mu} \tilde{\mathbf{T}}_{i_k^{(\ell)}} [\mathbf{I}_{N_s}; \mathbf{0}], \quad (10)$$

where $\tilde{\mu}$ specifies a scaling factor to gratify the power limitation and the columns of the $\tilde{\mathbf{T}}_{i_k^{(\ell)}}$ are the generalized eigenvectors of the pair $(\mathbf{H}_{i_k^{(\ell)}, i_k}^{\text{EffH}} \mathbf{H}_{i_k^{(\ell)}, i_k}^{\text{Eff}}, \tilde{\sigma}_{i_k^{(\ell)}}^2 \mathbf{I}_{M_t^{\text{RF}}} + \bar{\mathbf{H}}_{i_k^{(\ell)}}^{\text{intraH}} \bar{\mathbf{H}}_{i_k^{(\ell)}}^{\text{intra}})$.

Given the optimal baseband precoders by (10), the optimal

Algorithm 1 Hidden-node-aware SLNR-based Hybrid Transceiver Design

Require: Channel state information at the all transmission node, i.e., $i \in \{\mathcal{N}_{\text{NR}}, \mathcal{N}_{\text{WiGig}}\}$.

- 1: **for** $\ell = 1$ to N **do**
- 2: **for** $i = 1$ to $n_{\text{NR}} + n_{\text{WiGig}}$ **do**
- 3: **for** $k = 1$ to $|\mathcal{U}_i|$ **do**
- 4: Calculate the RF precoder $\mathbf{V}_{i_k^{(\ell)}}^{\text{RF}*}$ using (5) and (6).
- 5: Calculate the RF combiner $\mathbf{W}_{i_k^{(\ell)}}^{\text{RF}*}$ using (8) and (9).
- 6: Calculate the baseband precoder $\mathbf{V}_{i_k^{(\ell)}}^{\text{BB}*}$ using (10).
- 7: Calculate the baseband combiner $\mathbf{W}_{i_k^{(\ell)}}^{\text{BB}*}$ using (11).
- 8: **end for**
- 9: **end for**
- 10: **end for**

baseband combiners are designed as the well-known minimum mean square error (MMSE) receivers as follows

$$\hat{\mathbf{W}}_{i_k^{(\ell)}}^{\text{BB}*} = \mathbf{J}_{i_k^{(\ell)}}^{-1} \mathbf{H}_{i_k^{(\ell)}, i_k}^{\text{Eff}} \mathbf{V}_{i_k^{(\ell)}}^{\text{BB}*}, \quad (11)$$

where $\mathbf{J}_{i_k^{(\ell)}}$ denotes the covariance matrix of the total received signal at user $i_k^{(\ell)}$ and can be expressed as

$$\mathbf{J}_{i_k^{(\ell)}} = \sum_m \mathbf{H}_{i_k^{(\ell)}, i_m}^{\text{Eff}} \mathbf{V}_{i_m^{(\ell)}}^{\text{BB}*} \mathbf{V}_{i_m^{(\ell)}}^{\text{BBH}} \mathbf{H}_{i_k^{(\ell)}, i_m}^{\text{EffH}} + \tilde{\sigma}_{i_k^{(\ell)}}^2 \mathbf{I}. \quad (12)$$

We summarize the introduced algorithm in Algorithm 1.

V. SIMULATION RESULTS

In this preliminary numerical evaluation, we assess the performance of the proposed hidden-node-aware SLNR-based hybrid beamforming algorithm in a coexistence scenario. The setup of parameters is given in Table I. The propagation environment is modeled as in [13], i.e., a 8 cluster environment, each has 10 rays, with Laplacian distributed elevation and azimuth angle of arrival (AoAs) and angle of departure (AoDs). We implement the proposed algorithm along with the optimal unconstrained precoding and beam-steering solution. While the proposed hidden-node-aware SLNR-based hybrid beamforming algorithm mitigates both intra- and inter-cell interference, in the beam-steering solution method data streams are steered onto and received along the best channel propagation paths and the optimal unconstrained precoding method refers to sending data streams cross the channel's dominant eigenmodes. In comparison between methods, both single-stream communication, i.e., $N_s = 1$, and multi-stream communication, i.e., $N_s = 4$, are considered and discussed.

Fig. 1 demonstrates the cumulative distribution functions of the spectral efficiency that is achieved in a coexistence scenario where one NR-U gNB and one WiGig AP, each is equipped with planar arrays and serving two users, compete for spectrum sharing channel access. Fig. 1 shows that, in the medium to high SNR ranges, the proposed method achieves higher overall spectral efficiency and outperforms other methods owing to its inter- and intra-cell interference mitigation design. Moreover, in comparison to the traditional beam-steering solutions, there is a significant improvement that emphasizes the importance of designing advanced beamformers in mmWave coexistence scenarios. Note that even though beam-steering method is supposed to provide the most favorable result (in the absence

Table I: MAC and PHY layer parameters

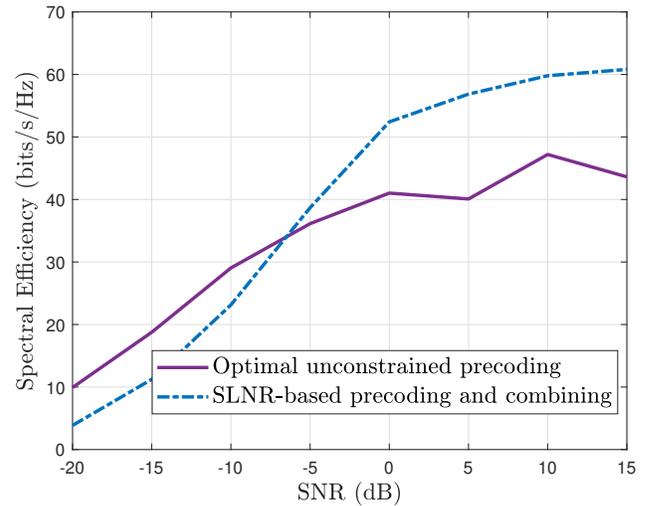
Parameter	Value	Parameter	Value
WiGig min contention window size	16	Number of transmit antennas	64
NR-U min contention window size	16	Number of receive antennas	16
WiGig max back-off stage	6	Number of RF chains at each transmitter	4
NR-U max back-off stage	3	Number of RF chains at each receiver	4
WiGig's packet payload duration	1 ms	Transmit power at each transmitter	35 dBm
NR-U's packet payload duration	1 ms	Noise figure at each receiver	9 dB
MAC header	272 bits	Thermal noise	-174 dBm/Hz
PHY header	128 bits	Transmitter antenna gain	8 dBi
ACK	112 bits + PHY header	Layout scenario	Indoor - office layout
Short inter-frame space (SIFS)	16 μ s	Path loss model	Free-space
Distributed inter-frame space (DIFS)	34 μ s	Number of channel realizations	100
Idle time slot	9 μ s	Array type	Uniform planar array

of large antenna arrays [13]), our proposed solution performs better than this method in mmWave frequencies when transmitters and receivers are supplied with large antenna arrays.

In Fig. 2 we consider a coexistence scenario in which four NR-U gNBs and four WiGig APs, each serving five users, share mmWave frequencies. The proposed algorithm outperforms the optimal unconstrained method, revealing its effectiveness in mitigating co-channel interference even for more complex multi-stream scenarios.

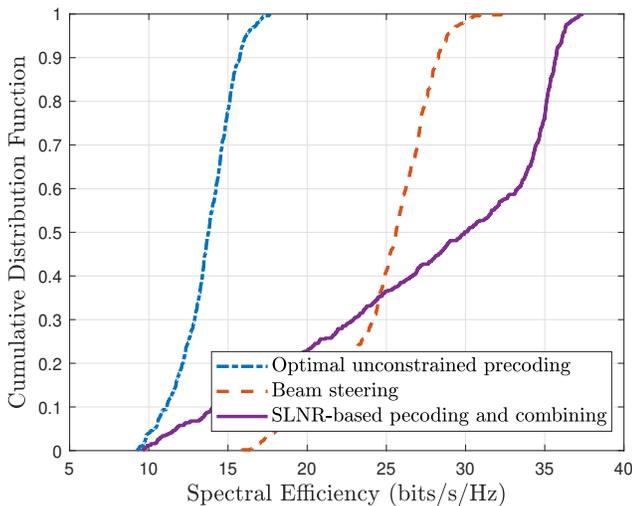
VI. CONCLUSION

In this work, we take a multi-cell, multi-user, massive MIMO coexistence scenario into account in which multiple 5G NR-U and WiGig links share an unlicensed millimeter-wave band. To increase the spectral efficiency of the unlicensed band, we have developed a hidden-node-aware hybrid precoding and combining method that mitigates both intra- and inter-cell interference. The hidden-node-aware SLNR-based algorithm outperforms both beam-steering and optimal unconstrained precoding, revealing its effectiveness in mitigating co-channel interference and the importance of designing the beamformers to enable coexistence of multiple networks.

Figure 2: Spectral efficiency achieved by different methods ($N_s = 4$)

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Figure 1: CDFs of the overall network spectral efficiency ($N_s = 1$)

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